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Solution of the plane stochastic creep boundary value problem☆

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ABSTRACT

The solution of the non-linear stochastic boundary-value problem of the creep of a thin plate in a plane stress state when the elastic strains are small and can be neglected is presented. The plate material is stochastically inhomogeneous so that the stress and strain tensors are random functions of the coordinates. The constitutive creep relation, taken as in non-linear viscous flow theory, is formulated in a stochastic form. Using the perturbation method, the non-linear stochastic problem is reduced to a system of three linear partial differential equations in the fluctuations of the stress tensor and, then, changing by implementing the stress function, to a differential equation, the solution of which is represented in the form of the sum of two series. The first series is the solution far from the boundary of the plate, ignoring edge effects, and the second is the solution in the boundary layer, and its terms rapidly decay as the distance from the boundary of the plate increases. The stretching of a stochastically inhomogeneous half-plane in the direction of two mutually orthogonal axes is considered as an example. The stress concentration in the boundary of the half-plane is investigated. It is shown that the spread of the stresses in the surface layer, the width of which depends on the degree of non-linearity of the material, can be much greater than in the deep layers.

Structural inhomogeneity in a material causes the appearance of a number of mechanical effects which cannot be investigated using classical phenomenological theories. One of these is the boundary-layer effect: close to the boundary of a body with structural inhomogeneity there is a boundary layer in which the stress-strain state differs from the stress-strain state of the internal regions. A stress concentration of occurs on the a body boundary which can attain an appreciable magnitude. This effect has been investigated in detail using the theory of stochastic functions, for linearly elastic media (see Refs 1–5 etc). Under creep conditions, the development of analytical methods for solving of stochastic boundary-value problems encounters serious difficulties, due mainly to physical and statistical non-linearities. Boundary effects under creep conditions based on the solution of a stochastic boundary-value problem, has therefore only been investigated in the simplest cases [6,7]. A plane stochastic steady-state creep problem, ignoring the boundary effect, has been solved [8] using the eigenvalue representation of a stochastic function; in this case, the boundary conditions were replaced by the requirement that the functions were bounded at infinity.

1. Formulation of the problem

Suppose the components of the stress tensor satisfy the equilibrium equations

$$\sigma_{ij,j} = 0, \quad i,j=1,2 \tag{1.1}$$

and the components of the strain rate tensor satisfy the condition

$$\Lambda_{ij}\Lambda_{kl}\dot{p}_{jk,il} = 0 \tag{1.2}$$

which is obtained from the strain compatibility equation by differentiation with respect to time. Here, Λ_{ij} are the components of the unit antisymmetric pseudotensor. Summation from 1 to 2 is carried out over repeated indices.

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Equations. (1.1) and (1.2) are closed by the constitutive relation, taken, as in non-linear viscous flow theory, in stochastic form [8]:

$$\dot{p}_{ij} = cs^{n-1}\left(\sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}\right)(1+\alpha U); \quad s^2 = \frac{1}{2}(3\sigma_{ij}\sigma_{ij} - \sigma_{ii}\sigma_{jj}) \tag{1.3}$$

where S is the stress intensity, δ_{ij} is the Kronecker delta, $U(x_1,x_2)$ is a homogeneous stochastic function which describes the fluctuations of the rheological properties of a material with mathematical expectation $\langle U\rangle=0$ and dispersion $\langle U^2\rangle=1$, c and n are material constants and α is the degree of inhomogeneity of the material.

Determinate surface forces q_i are specified on the contour Γ of the domain S occupied by the plate:

$$\sigma_{ij}n_j\big|_\Gamma = q_i \tag{1.4}$$

where n_i are the components of the unit vector of the normal to the contour Γ.

Relations (1.1)–(1.3), with boundary conditions (1.4), specify a stochastic creep problem which is approximately solved below for the stresses.

2. Solution of the problem for a plate

Suppose the components of the stress and strain tensors are represented in the form of the sum of a determinate term and a fluctuation:

$$\begin{aligned} &\sigma_{ij} = \sigma_{ij}^0 + \sigma_{ij}^*, \quad \langle\sigma_{ij}\rangle = \sigma_{ij}^0, \quad \left\langle\sigma_{ij}^*\right\rangle = 0 \\ &p_{ij} = p_{ij}^0 + p_{ij}^*, \quad \langle p_{ij}\rangle = p_{ij}^0, \quad \left\langle p_{ij}^*\right\rangle = 0 \end{aligned} \tag{2.1}$$

The tensors with the components σ_{ij}^0 and p_{ij}^0 are assumed to be known and can be found as the solution of the corresponding determinate steady-state creep problem.

Relation (1.3) is linearized statistically with respect to the fluctuations σ_{ij}^* taking account of the possibility of neglecting products of the form $\sigma_{ij}^*\sigma_{kl}^*$, $\alpha U\sigma_{ij}^*$. With the aim of physical linearization, the function s^{n-1}, appearing in the constitutive creep relation (1.3), is expanded in a power series and only the linear terms in this expansion are taken into account:

$$s^{n-1} = \left(s_0^2 + s_*\right)^m \approx s_0^{n-1} + ms_0^{n-3}s_* = s_0^{n-1}(1+ks_*) \tag{2.2}$$

Here,

$$\begin{aligned} &s_0^2 = \sigma_{11}^{0\,2} + \sigma_{22}^{0\,2} - \sigma_{11}^0\sigma_{22}^0 + 3\sigma_{12}^{0\,2}, \quad s_* = l_1\sigma_{11}^* + l_2\sigma_{22}^* + 6\sigma_{12}^0\sigma_{12}^* \\ &l_1 = 2\sigma_{11}^0 - \sigma_{22}^0, \quad l_2 = 2\sigma_{22}^0 - \sigma_{11}^0, \quad k = \frac{m}{s_0^2}, \quad m = \frac{n-1}{2} \end{aligned}$$

According to relations (1.3), (2.1) and (2.2), the fluctuations of the rate of strain tensor with the components p_{ij}^* have the form

$$\dot{p}_{\beta\beta}^* = \frac{1}{3}cs_0^{n-1}\left(3\sigma_{\beta\beta}^* - \sigma_{ii}^* + ks_*l_\beta + \alpha Ul_\beta\right), \quad \dot{p}_{12}^* = cs_0^{n-1}\left(\sigma_{12}^* + ks_*\sigma_{12}^0 + \alpha U\sigma_{12}^0\right), \tag{2.3}$$

(no summation over the subscript β).

If expressions (2.3) are substituted into the compatibility equation for the strain rate fluctuations $\Lambda_{ij}\Lambda_{kl}\dot{p}_{jk,il}^* = 0$, a linear partial differential equation with variable coefficients with respect to σ_{ij}^* can be obtained. Since it is difficult to solve this equation, we will restrict the treatment to the class of problems in which σ_{ij}^0 are constant quantities. The linear boundary-value problem in σ_{ij}^* then takes the form

$$\begin{aligned} &\sigma_{ij,j}^* = 0 \\ &\left(2+kl_1^2\right)\sigma_{11,22}^* + (-1+kl_1l_2)\left(\sigma_{11,11}^* + \sigma_{22,22}^*\right) + \left(2+kl_2^2\right)\sigma_{22,11}^* \\ &+ 6k\sigma_{12}^0\left(l_1\left(\sigma_{12,22}^* - \sigma_{11,12}^*\right) + l_2\left(\sigma_{12,11}^* - \sigma_{22,12}^*\right)\right) - 6\left(1+6k\sigma_{12}^{0\,2}\right)\sigma_{12,12}^* \\ &= -\alpha(l_1U_{,22} + l_2U_{,11} - 6\sigma_{12}^0U_{,12}) \\ &\sigma_{ij}^*n_j\big|_\Gamma = 0 \end{aligned} \tag{2.4}$$

If the stress function F for the fluctuations in the stress tensor is introduced using the formulae

$$\sigma_{11}^* = F_{,22}, \quad \sigma_{22}^* = F_{,11}, \quad \sigma_{12}^* = -F_{,12} \tag{2.5}$$

then, instead of system of equations (2.4), we obtain a single differential equation in the function F

$$\begin{aligned} &\left(2+kl_2^2\right)F_{,1111} + \left(2+kl_1^2\right)F_{,2222} - 12\sigma_{12}^0k(l_1F_{,1222} + l_2F_{,1112}) \\ &+ 2F_{,1122}\left(2+kl_1l_2+18k\sigma_{12}^{0\,2}\right) = -\alpha(l_1U_{,22} + l_2U_{,11} - 6\sigma_{12}^0U_{,12}) \end{aligned} \tag{2.6}$$

with boundary conditions

$$F_{,22}n_1 - F_{,12}n_2\big|_\Gamma = 0, \quad -F_{,12}n_1 + F_{,11}n_2\big|_\Gamma = 0 \tag{2.7}$$

Suppose the homogeneous function $U(x_1,x_2)$, by using which the random field of the perturbations in the rheological properties of the material is specified, is an almost periodic, rapidly oscillating function of the coordinates [1]:

$$U = \sum_{k=1}^{\infty} A_k \cos(\omega c_k x_1 + \omega d_k x_2 + \varphi_k) \tag{2.8}$$

where ω is a parameter with the dimension of inverse length, c_k and d_k are dimensionless quantities of the order of unity, A_k are centred equally distributed random quantities, and φ_k are random quantities, uniformly distributed in the interval (0, 2π), and, moreover, all the random quantities A_k and φ_k are independent.

For convenience, it is useful to change to the function of a complex variable

$$\tilde{U} = \sum_{k=1}^{\infty} \tilde{A}_k \exp(i\omega(c_k x_1 + d_k x_2)); \quad \tilde{A}_k = A_k \exp(i\varphi_k) \tag{2.9}$$

The function $\tilde{U}$ is introduced such that $\mathrm{Re}\tilde{U} = U$.

Consider the boundary-value problem (2.6), (2.7), in which U and F are replaced by the quantities $\tilde{U}$ and $\tilde{F}(\mathrm{Re}\tilde{F} = F)$. The solution can be represented in the form

$$\tilde{F} = \sum_{k=1}^{\infty} (\upsilon_k + w_k) \tag{2.10}$$

Here, υ_k is a special solution of Eq. (2.6) in which the function $\tilde{U}$ is replaced by the k-th term of expansion (2.9), and w_k is the solution of the homogeneous equation, corresponding to (2.6), which satisfies the conditions on the boundary Γ

$$\begin{aligned} &\upsilon_{k,22}n_1 - \upsilon_{k,12}n_2\big|_\Gamma = -w_{k,22}n_1 + w_{k,12}n_2\big|_\Gamma \\ &-\upsilon_{k,12}n_1 + \upsilon_{k,11}n_2\big|_\Gamma = w_{k,12}n_1 - w_{k,11}n_2\big|_\Gamma \end{aligned} \tag{2.11}$$

We shall seek the function υ_k in the form

$$v_k = f_k \exp(i\omega(c_k x_1 + d_k x_2)), f_k = \mathrm{const} \tag{2.12}$$

An algebraic equation is then obtained for finding f_k, from which it follows that

$$f_k = \frac{\alpha\tilde{A}_k(m_k - 6c_k d_k \sigma_{12}^0)}{\omega^2\left(2\left(c_k^2 + d_k^2\right)^2 + km_k^2 + 12kc_k d_k \sigma_{12}^0\left(3c_k d_k \sigma_{12}^0 - m_k\right)\right)}, \quad m_k = d_k^2 l_1 + c_k^2 l_2$$

The series $\sum_{k=1}^{\infty} w_k$ defines a boundary-layer type solution which rapidly decays with depth into the body. We will construct, for example, a solution w_k of the boundary-layer type in the domain $x_2 \geq -b$ close to the body boundary $x_2 = -b$. Making the replacement

$$w_k = g_k(t)\exp(i\omega(c_k x_1 - d_k b)), \quad t = \omega(b + x_2) \tag{2.13}$$

for the function $g_k(t)$, we obtain the following differential equation

$$\begin{aligned} &(2 + kl_1^2)g_k'''' - 12ic_k k\sigma_{12}^0 l_1 g_k''' - 2c_k^2\left(2 + kl_1 l_2 + 18k\sigma_{12}^{0\,2}\right)g_k'' \\ &+ 12ikc_k^3\sigma_{12}^0 l_2 g_k' + c_k^4\left(2 + kl_2^2\right)g_k = 0 \end{aligned} \tag{2.14}$$

with boundary conditions

$$g_k(0) = -f_k, \quad g_k'(0) = -id_k f_k \tag{2.15}$$

A prime denotes a derivative with respect to t.

Equation (2.14) was obtained by substituting expressions (2.13) into the homogeneous equation corresponding to (2.6), and the boundary conditions were obtained from condition (2.11) using expressions (2.12) and (2.13).

The solution of Eq. (2.14), when all the roots r_s^k of the corresponding characteristic equation are simple, has the form

$$g_k(t) = \sum_{s=1}^{4} C_s^k \exp\left(r_s^k t\right) \tag{2.16}$$

where C_s^k are arbitrary constants.

3. Solution of the problem for a half-plane

As an example, we will consider the creep of a stochastically inhomogeneous half-plane $x_2 \geq 0$ under plane stress state conditions. Suppose loads

$$\sigma_{22}|_{x_2=0} = \sigma_{22}^0 = \text{const}, \quad \sigma_{12}|_{x_2=0} = 0$$

are applied to the boundary of the half-plane $x_2 = 0$ and the stress σ_{11} satisfies the macroscopic homogeneity condition $\langle\sigma_{11}\rangle = \sigma_{11}^0$ which corresponds to the application of a load at infinity ($x_1 \to \pm\infty$).

We will construct a solution of the boundary-layer type close to the boundary of the half-plane $x_2 \geq 0$.

The solution of Eq. (2.14) under the conditions

$$\sigma_{11}^0 \neq \sigma_{22}^0, \quad \sigma_{12}^0 = 0 \tag{3.1}$$

is given by formula (2.16) and, here, the roots r_s^k of the characteristic equation are given by the expressions

$$r_{1,2}^k = -c_k(B_+ \pm iB_-), \quad r_{3,4}^k = c_k(B_+ \pm iB_-)$$
$$B_\pm = \frac{\sqrt{\pm a + \sqrt{a^2 + b^2}}}{\sqrt{2}}, \quad a = \frac{2 + kl_1l_2}{2 + kl_1^2}, \quad b = \frac{\sqrt{2k}|l_1 - l_2|}{2 + kl_1^2}$$

Of the four roots of the characteristic equation, two roots r_3^k and r_4^k have positive real parts. Since the boundary effect must decay when $x_2 \to \infty$, the two constants C_3^k and C_4^k, corresponding to these roots, are equal to zero. Boundary conditions (2.15) are used to find the other two constants C_1^k and C_2^k.

The solution of Eq. (2.14), of the boundary-layer type under conditions (3.1), has the form

$$g_k(t) = f_k\left[\frac{r_1^k - id_k}{r_2^k - r_1^k}\exp(r_2^k t) - \frac{r_2^k - id_k}{r_2^k - r_1^k}\exp(r_1^k t)\right] \tag{3.2}$$

Under the condition $\sigma_{11}^0 = \sigma_{22}^0 = \sigma^0$, there are multiple roots of the characteristic equation. The solution of this problem has been presented earlier [6] and is not considered here.

Substituting expressions (2.12), (2.13) and (2.16) into relation (2.10), we find the stress function

$$\tilde{F} = \sum_{k=1}^{\infty} \exp(ic_k\omega x_1)(f_k \exp(id_k\omega x_2) + g_k(x_2))$$

Separating out the real part and introducing the notation

$$\chi_\pm(x_2) = \xi_k(x_2)(B_-\cos(c_kB_-\omega x_2) \pm B_+\sin(c_kB_-\omega x_2))$$
$$\xi_k(x_2) = \frac{\exp(-c_kB_+\omega x_2)}{B_-}, \quad h_k = \frac{m_k}{2\left(c_k^2 + d_k^2\right)^2 + km_k^2}, \quad B^2 = B_+^2 + B_-^2$$

we obtain

$$F = \operatorname{Re}\tilde{F} = \frac{\alpha}{\omega^2}\sum_{k=1}^{\infty} A_kh_k\{\cos(c_k\omega x_1 + \varphi_k)[\cos(d_k\omega x_2) - \chi_+(x_2)]$$
$$-\sin(c_k\omega x_1 + \varphi_k)\left[\sin(d_k\omega x_2) - \frac{d_k}{c_k}\xi_k(x_2)\sin(c_kB_-\omega x_2)\right]\} \tag{3.3}$$

Table 1

n	h=0	0.25	0.5	2	4	8
3	1.22	1.31	1.48	2.35	2.07	1.89
5	1.00	1.11	1.32	2.65	2.13	1.82
7	0.89	1.02	1.24	2.92	2.17	1.77
9	0.82	0.96	1.20	3.16	2.20	1.74

According to formulae (2.5), the stresses corresponding to the stress function (3.3) are given by the expressions

$$\sigma_{11}^{*} = \alpha\sum_{k=1}^{\infty} A_k h_k \Big\{\cos(c_k\omega x_1+\varphi_k)\Big[-d_k^2\cos(d_k\omega x_2)+c_k^2B^2\chi_-(x_2)\Big]$$
$$+\sin(c_k\omega x_1+\varphi_k)\Big[d_k^2\sin(d_k\omega x_2)+c_kd_k\xi_k(x_2)$$
$$\times\Big(B^2\sin(c_kB_-\omega x_2)-2B_+B_-\cos(c_kB_-\omega x_2)\Big)\Big]\Big\}$$

$$\sigma_{22}^{*} = \alpha\sum_{k=1}^{\infty} A_k h_k c_k^2\{-\cos(c_k\omega x_1+\varphi_k)[\cos(d_k\omega x_2)-\chi_+(x_2)]$$
$$+\sin(c_k\omega x_1+\varphi_k)\cdot\left(\sin(d_k\omega x_2)-\frac{d_k}{c_k}\xi_k(x_2)\sin(c_kB_-\omega x_2)\right)\Big\}$$

$$\sigma_{12}^{*} = \alpha\sum_{k=1}^{\infty} A_k h_k c_k\{\sin(c_k\omega x_1+\varphi_k)\Big[-d_k\sin(d_k\omega x_2)+c_k\xi_k(x_2)B^2$$
$$\times\sin(c_kB_-\omega x_2)]+d_k\cos(c_k\omega x_1+\varphi_k)[\cos(d_k\omega x_2)-\chi_-(x_2)]\}$$

We will calculate the dispersions of the stochastic stress field $D_{ij}=\langle|\sigma_{ij}^{*}|^2\rangle$ assuming the values of c_k and d_k to be equal to unity. Under these condition, the stochastic field U, defined by expansion (2.8), can be assumed to be approximately isotropic. [1] Taking account of the conditions imposed on the stochastic quantities A_k and φ_k and the equality $\langle U^2\rangle$=1, the dispersions of the stochastic stress field are given by the following expressions

$$D_{11}(x_2)=\Lambda\Big\{\Big[-\cos\omega x_2+B^2\chi_-(x_2)\Big]^2$$
$$+\Big[\sin\omega x_2+\xi(x_2)\Big(B^2\sin(B_-\omega x_2)-2B_+B_-\cos(B_-\omega x_2)\Big)\Big]^2\Big\}$$
$$D_{22}(x_2)=\Lambda\Big\{[\cos\omega x_2-\chi_+(x_2)]^2+[\sin\omega x_2+\xi(x_2)\sin(B_-\omega x_2)]^2\Big\}$$
$$D_{12}(x_2)=\Lambda\Big\{[\cos\omega x_2-\chi_-(x_2)]^2+\Big[-\sin\omega x_2+B^2\xi(x_2)\sin(B_-\omega x_2)\Big]^2\Big\} \quad (3.4)$$

where

$$\Lambda=\frac{8\alpha^2s_0^2(l_1+l_2)^2}{\left(16s_0^2+(n-1)(l_1+l_2)^2\right)^2},\quad \xi(x_2)=\frac{\exp(-B_+\omega x_2)}{B_-}$$

The results obtained enable us to analyse the main features of the effect of a boundary layer when there is creep.

According to relation (3.4), the dispersion of the stress σ_{11}^{*} on the boundary of the half plane $x_2=0$ and when $x_2\to\infty$ is given by the formulae

$$D_{11}(0)=\Lambda\Big(1+a^2+2a+b^2\Big),\quad D_{11}(\infty)=\Lambda$$

The stress concentration on the boundary of the half-plane $x_2=0$ is characterized by the coefficient of variability of the root-mean-square deviation

$$\rho=\sqrt{\frac{D_{11}(0)}{D_{11}(\infty)}}=\sqrt{1+a^2+2a+b^2}$$

Values of the coefficient of variability ρ are presented in Table 1 as a function of the degree of non-linearity of the steady-state creep n and the loading parameter $h=\sigma_{22}^0/\sigma_{11}^0$; $\rho=2$ for $n=1$ and any h, and, also, for $h=1$ and any n.

Table 2

n	α=0.05	0.1	0.2	0.5
1	5.30	10.61	21.21	53.03
3	4.52	9.04	18.09	45.23
5	4.01	8.02	16.05	40.09
7	3.64	7.28	14.56	36.38
9	3.35	6.71	13.42	33.54

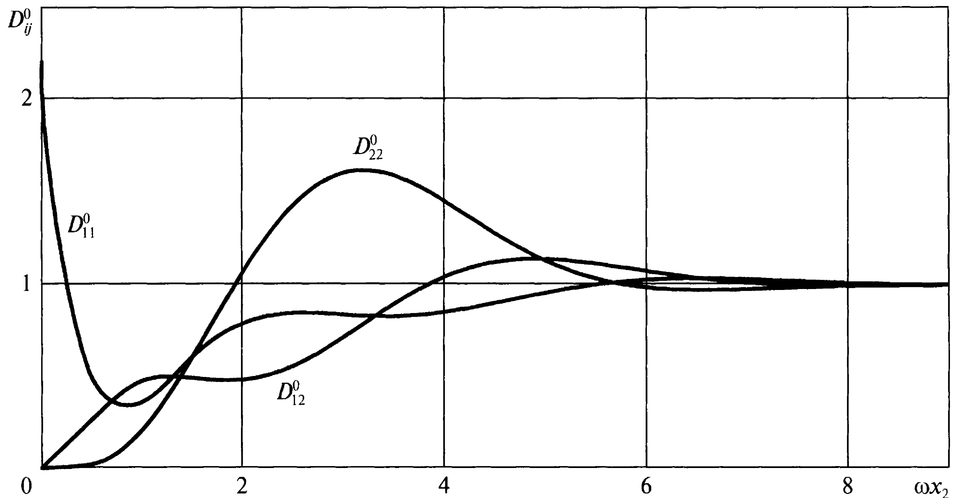

Fig. 1. .

The values of the coefficient of variation $d_{11} = (\sqrt{D_{11}(0)}/\sigma_{11}^0) \times 100\%$ on the boundary of the half-plane $x_2 = 0$ and $h = 2$ are presented in Table 2 as a function of the variables α and n. It follows from Table 1 that the quantity d_{11} has its maximum value for this value of h (for fixed α and n).

In the case of materials with a high non-linearity exponent ($n = 9$), it can be seen from Table 2 that the coefficient of variation lies in the range from 3.35% (α = 0.05) to 33.5% (α = 0.5).

In the case of low non-linearity exponents, when the creep law can be linearized ($n = 1$), the scatter of the stresses σ_{11}^* around the mean value is greater and, here, the value of d_{11} lies in the range from 5.3 to 53%.

The normalized dispersions of the stresses $D_{ij}^0(D_{ij}^0 = D_{ij}(x_2)/D_{ij}(\infty))$ as a function of the dimensionless coordinate ωx_2 for $n = 3$ and $h = 0.5$ are shown in Fig. 1. The normalized dispersions D_{12}^0 and D_{22}^0 on the boundary of the half plane are equal to zero and $D_{11}^0 = 2.2$. As x_2 increases, the dispersions converge quite rapidly to constant values, which are identical to their values for an unbounded medium. When $\omega x_2 \geq 6$, the relative error arising from the replacement of the normalized dispersions by unity does not exceed 5%. It can therefore be assumed that the boundary of the edge effect zones is numerically identical to the quantity 6/ω.

It was established in the investigation of the normalized dispersions for different values of the non-linearity exponent n and the loading parameter h that, as the parameter h increases for fixed values of n, the boundary layer zone contracts. The normalized dispersion D_{11}^0 has a maximum value on the boundary of the half-plane, and D_{12}^0 and D_{22}^0 have a maximum value in the boundary layer while, when n increases (for a fixed h), the maximum value of D_{11}^0 decreases and D_{12}^0 and D_{22}^0 increase. The parameter n has no significant effect on the boundary layer thickness. The non-monotonicity of the graphs for the stress dispersions is due to the fact that the random microinhomogeneities are simulated using an almost periodic rapidly oscillating function of the coordinates.

Hence, the stress fluctuations in the surface layer reach values which can be significantly greater than those in the deep layers. It is clear from this that the fluctuations in the boundary-layer stress play an important role in resolving the question of the reliability of structural components using long-term endurance and instantaneous local stress criteria because of the variations of these stresses. The failure to take account of boundary effects can lead to an unfounded overestimation of the fatigue life of structural components under creep conditions.

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